October Math Gems

Problem of the week 6

§1 Problems

Problem 1.1. Solve the following equation

$$(\sqrt{2+\sqrt{3}})^x + (\sqrt{2-\sqrt{3}})^x = 2^x$$

Problem 1.2. Solve for x

$$\frac{4^{2x}+4^x+1}{2^{2x}+2^x+1} = 13$$

Problem 1.3. If $\sin A + \sin^2 A = 1$ and $a\cos^{12} A + b\cos^8 A + c\cos^6 A - 1 = 0$, then the value of

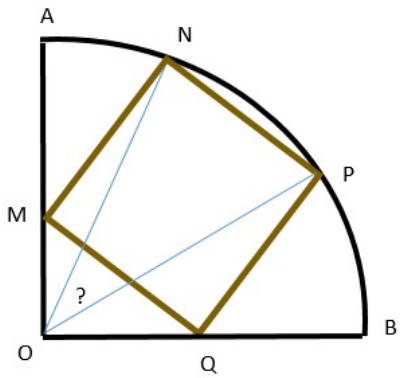
$$b + \frac{c}{a} + b$$

is?

Problem 1.4. If $x + \frac{1}{x} = 1$, Find the value of

$$x^{21} + x^{18} + x^{12} + x^9 + x^3 + 1$$

Problem 1.5. AOB is a quadrant and $MNPQ$ is a square . Find the value of the unknown angle.

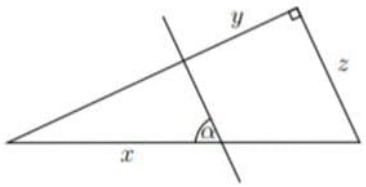


Problem 1.6. Solve for x

$$\frac{1}{1-\sqrt{1-x}} - \frac{1}{1+\sqrt{1-x}} = \frac{\sqrt{3}}{x}$$

Problem 1.7. Prove or disprove: $\forall$ integer values of x, then $x^9 - 6x^7 + 9x^5 - 4x^3$ is divisible by 8640.

Problem 1.8. The drawing below shows a right-angled triangle. A straight line crosses the triangle parallel to the line z and encloses an angle of α. The lengths x and y of the bottom and top line segments as well as the angle α are given. Find an equation for the length z.



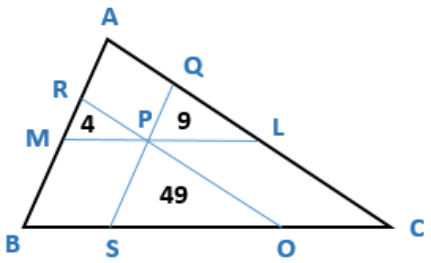
Problem 1.9. Solve this system of equations

$$\begin{cases} \sqrt{x+y} = 72 - x - y \\ \sqrt{x-y} = x - y - 30 \end{cases}$$

Problem 1.10. Find the value of x

$$x^{x^6} = \sqrt{2}^{\sqrt{2}}$$

Problem 1.11. P is in the interior of $\triangle ABC$, lines trough P parallel to the sides of $\triangle ABC$, the resulting smaller triangles have areas $t_1 = 4, t_2 = 9, t_3 = 49$. Find the area of $\triangle ABC$.



Problem 1.12. Solve this system of equations

$$\begin{cases} x^2 = y^3 + 1 \\ y^2 = x^3 - 23 \end{cases}$$

Problem 1.13. Solve for x

$$(x+2)^2 + (x+3)^3 + (x+4)^4 = 2$$

Problem 1.14. Find a, b

$$x^4 - 4x^3 + ax^2 + bx + 1 = 0$$

Problem 1.15.

$$8^a = 27^b = 125^c = 30, \qquad \frac{abc}{ab+bc+ca} =?$$

Problem 1.16. If $x - 5\sqrt{x} - 1 = 0$, Find the value of

$$x^2 + \frac{1}{x^2}$$

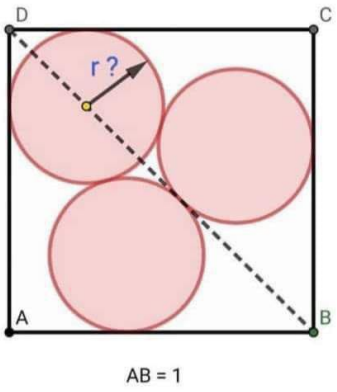
Problem 1.17. If $a + \frac{1}{a} = 5$, Find the value of

$$\sqrt{\frac{(a^5+a^3)(a^3+a)}{4a^6}}$$

Problem 1.18. If $2f(x) + f(1-x) = x^2 \forall x$, then find

$$f(x) =?$$

Problem 1.19. Three circles with the same radius r are inscribed in a square that has a length of 1. Find the length of the radius.



Problem 1.20.

$$(x + x^2 + x^3) + (\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}) = 28$$

Find the value of

$$(2x-3)^2$$